The advantages of the proposed test in interval estimation are self-evident. For instance, the extreme points of the 100(1 - q)-percent confidence interval for the coefficients $\hat{\beta}_j$ should be computed from the formula

$$
\hat{\beta}_j - \Delta \hat{\beta}_j \pm \frac{t \left( \tilde{S} \left( 1 - q/2 \right) S V \tilde{C} \right)}{\sqrt{n - k + \tilde{\nu}}},
$$

where $i = 1, \ldots, k - 1$ and correspondingly the predictor of the dependent variable for a fixed $X^* = (x^*_1, \ldots, x^*_k)$ should be computed from the formula

$$
\sum_{j=1}^{k} \left( \hat{\beta}_j - \Delta \hat{\beta}_j \right) \pm \frac{t \left( \tilde{S} \left( 1 - q/2 \right) S V (\tilde{X}^* \tilde{X}^*)^{-1} \tilde{X}^* + \tilde{I} \right)}{\sqrt{n - k + \tilde{\nu}}},
$$

In conclusion note that the proposed method may be applied to extend some other tests (e.g., the Fisher test [5]) to the case of errors in measurements of independent variables.

**LITERATURE CITED**


**SIMULATING DENDRITIC-STRUCTURE FORMATION IN THE CRYSTALLIZATION OF A SUPERCOOLED LIQUID**

A. R. Umantsev, V. V. Vinogradov, and V. T. Borisov

The characteristics of a metal are largely determined by the parameters of the dendritic structure formed during crystallization; there are fairly many experimental studies on this [1-4]. However, major mathematical difficulties mean that theoretical studies are restricted to the growth of parabolic crystals [5-7]. Attempts have been made to derive analytic expressions for crystal surface shapes, which have given cumbersome integrodifferential equations even for stationary growth [8, 9], which are largely impossible to analyze. So far, there has been no theoretical description at all for the formation of secondary branches at the sides of a dendritic crystal.

These disadvantages can be overcome in simulating the process.

The simulation has been based on a two-dimensional treatment. We split up the region of space occupied by the supercooled liquid by means of a two-dimensional network $x, z, t = \{x_i = i \Delta x, i = 0, 1, \ldots, I; z_j = j \Delta z, j = 0, 1, \ldots, J; t_k = k \Delta t, k = 0, 1, \ldots\}$ giving cells of size $\Delta x \Delta z$, where $x$ and $z$ are the Cartesian coordinates (Fig. 1) and $t$ is time. The state of each cell is characterized by the mean temperature $T^k_{i,j}$ and the proportion of solid phase $\varphi^k_{i,j}$, which is the ratio of the volume of the crystalline material to the volume of the entire cell. The latter is 1 for cells completely filled with solid and is 0 for cells filled with liquid, while taking values $0 < \varphi^k_{i,j} < 1$ for solidifying cells, i.e., ones containing the phase boundary (Fig. 1).

In time $\Delta t$, the proportion of solid phase in a solidifying cell alters by $\Delta g_{ij}^k$, and heat $L \Delta x \Delta z \Delta g_{ij}^k$ is released, where $L$ is the latent heat of crystallization of unit volume. The cell heat-balance equation is

$$C \Delta x \Delta z \left( T_{ij}^{k+1} - T_{ij}^k \right) = \lambda \Delta t \left[ \frac{\Delta z}{\Delta x} \left( T_{i+1,j}^k + T_{i-1,j}^k - 2T_{ij}^k \right) + \frac{\Delta x}{\Delta z} \left( T_{i,j+1}^k + T_{i,j-1}^k - 2T_{ij}^k \right) \right] + L \Delta x \Delta z \Delta g_{ij}^k,$$  \hspace{1cm} (1)

where $C$ and $\lambda$ are the thermal capacity of unit volume and the thermal conductivity of the liquid, the values being taken as identical for the liquid and solid phases. The heat balance for a nonsolidifying cell obeys (1) with $\Delta g_{ij}^k = 0$. The change in $g_{ij}^k$ in a solidifying cell in time $\Delta t$ is

$$\Delta g_{ij}^k = V_n \frac{\Delta t}{\Delta x \Delta z}.$$

If the crystal grows by the normal mechanism [10], the rate $V_n$ of advance in the front is related to the temperature by

$$V_n = \mu (T_{cr} - T_0),$$

where $\mu$ is the kinetic growth coefficient and $T_{cr}$ is the crystallization temperature. It can be shown that the effects from the surface energy are here slight, and the only physical parameter governing the evolution is the relative supercooling $\Delta \theta = C(T_{cr} - T_0)/L$, where $T_0$ is the initial temperature of the supercooled liquid.
At the start \((k = 0)\), one specifies the initial state; to transfer from time step \(k\) to \(k + 1\), one considers all internal cells in sequence. It can be shown that the result is independent of the sequence. One first determines the changes in the proportions of solid, which are given by (2) for solidifying cells and are zero for nonsolidifying ones. Then from (1) one calculates \(T_{ij}^{k+1}\) in a cell in the next time step.

The simulation showed that a crystalline seed placed in the supercooled liquid is converted to an acicular crystal in a certain time. We examined the dependence of the vertex growth rate and curvature on the relative supercooling \(\Delta \theta\). The side surface gives rise to a system of equally spaced branches growing perpendicular to the main column (Fig. 2). The distances between branches are dependent on the initial supercooling. Thermal interaction between branches leads to the structure becoming coarser (Fig. 2). Here every second branch is retarded in growth and is ultimately trapped by the adjacent, more rapidly growing one. The distances between branches then double. In the later stages, the competition between the residual branches produces distance doubling again. The structure thus develops by regular distance doubling.

This branch evolution mechanism indicated by simulation is confirmed by photomicrographs of dendritic crystals for pure substances and alloys. Figures 9a and 10 of [2] indicate the successive stages in enlargements in the branches in succinonitrile, which is a metal-type organic compound. All the stages involve branch distance doubling. The same occurs in Sn + 60% Pb near the region of maximal diffusion-dependent supercooling of the liquate and dendrites of TiS in Fe-Ni-Cr alloy [11]. Templates from the columnar zone of a casting of Kh23N18 steel have also shown the above branch evolution mechanism.
Studies have been made [12] on the dendritic structures in ammonium chloride crystals. Figure 3 shows a dendrite grown from supersaturated aqueous solution. Near the top, there is a close-packed system of equally spaced branches. Far from the top, the structure enlarges by distance doubling. The grown crystal was then isothermally annealed, i.e., kept under conditions corresponding to equilibrium between the solid and liquid phases. The driving force under these conditions is the tendency of the excess surface energy to a minimum [2, 12]. Therefore, it is incorrect to assume, as is often done in the literature, that there are identical mechanisms for side-branch enlargement during growth and during isothermal annealing.

The simulation results also confirmed another feature. At high supercooling ($\Delta T \propto 1$), the crystal morphology gradually alters: the dendrites lose their lateral structure and acicular crystals grow into the liquid. For $\Delta T \gg 1.5$, the dendritic structure vanishes completely, and rounded crystals grow (Fig. 4). This may be called the globular transition, which has been repeatedly observed in organic substances [3, 4] and metals [13], but which has not received a physical explanation. The same phenomenon is evidently responsible for the dendrite-free structure in the surface zone of a metal casting, which crystallizes with the largest supercooling.

LITERATURE CITED

12. M. O. Kliya, Kristalllografiya, 1, No. 5, 577-582 (1956).