

## Lesson 9 Hypothesis Testing

### Outline

Logic for Hypothesis Testing

Critical Value

Alpha ( $\alpha$ )

-level .05

-level .01

One-Tail versus Two-Tail Tests

-critical values for both alpha levels

### **Logic for Hypothesis Testing**

Anytime we want to make comparative statements, such as saying one treatment is better than another, we do it through hypothesis testing. Hypothesis testing begins the section of the course concerned with inferential statistics. Recall that inferential statistics is the branch of statistics in which we make inferences about populations from samples. Up to this point we have been mainly concerned with describing our distributions using descriptive statistics.

Hypothesis testing is all about populations. Although we will start using just one value from the population and eventually a sample of values in order to test hypotheses, keep in mind that we will be inferring that what we observe with our sample is true for our population.

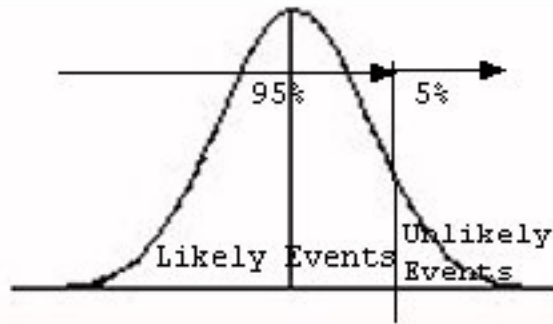
We will want to see if a value or sample comes from a known population. That is, if I were to give a new cancer treatment to a group of patients, I would want to know if their survival rate, for example, was different than the survival rate of those who do not receive the new treatment. What we are testing then is whether the sample patients who receive the new treatment come from the population we already know about (cancer patients without the treatment). Again, even though we are talking about a sample, we infer that the sample is just part of an entire population. The population is either the one we already know about, or some new population (created by the new treatment in this example).

### Logic

- 1) To determine if a value is from a known population, start by converting the value to a z-score and find out how likely the score is for the known population.
- 2) If the value is “likely” for the known population then it is likely that it comes from the known population (the treatment had no effect).
- 3) If the value is “unlikely” for the known population then it is probably does not come from the population we know about, but instead comes from some other unknown population (the treatment had an effect).

4) A value is “unlikely” if it is less than 5% likely for the known population. Any value that occurs 5% or more of the time for the known population is “likely” and part of the known population.

The 5% cut-off point is rather arbitrary, and it will change as we progress. For now we will use it as a starting point to illustrate several concepts.



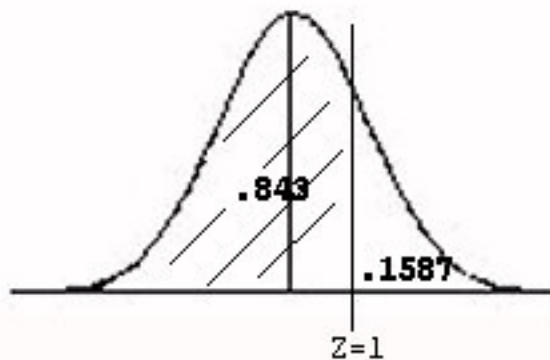
Let’s look at a simple example. Say the earth has been invaded by aliens that look just like humans. The only way to tell them apart from humans is to give them an IQ test since they are quite a bit smarter than the average human. Let’s say the average human IQ (the known population) is  $\mu = 100$   $\sigma = 15$ .

We want to know if Bob is an alien. Bob has an IQ score of 115? Is it likely that he comes from the known population and is human, or does he come from a different alien population? To answer the question, first compute the z-score.

$$Z = \frac{115 - 100}{15} = \frac{15}{15} = 1$$

Next, find out how likely this z-score is for the population.

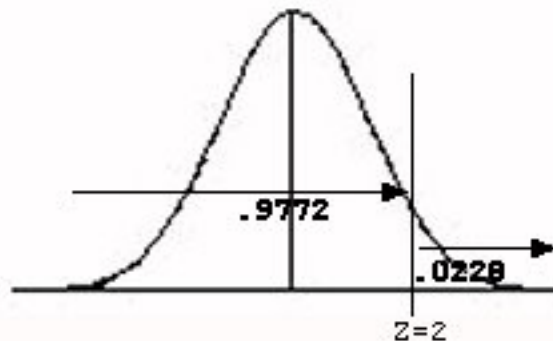
For hypothesis testing we will always be interested in whether the value is extreme for the population, or unlikely. Thus, we will be looking in the Tail Column when deciding if the value is “unlikely.”



So, the likelihood of observing an IQ of 115 or more is .1587. Since the probability is not less than 5% or .05 we have to assume Bob comes from the general population of humans.

Say Neil has an IQ of  $X = 30$ . Is it likely Neil comes from the general population? Again we first compute the z-score, and then find how likely it is to get that value or one more extreme.

$$Z = \frac{130 - 100}{15} = \frac{30}{15} = 2 \quad \text{Next, find out how likely this z-score is for the population.}$$



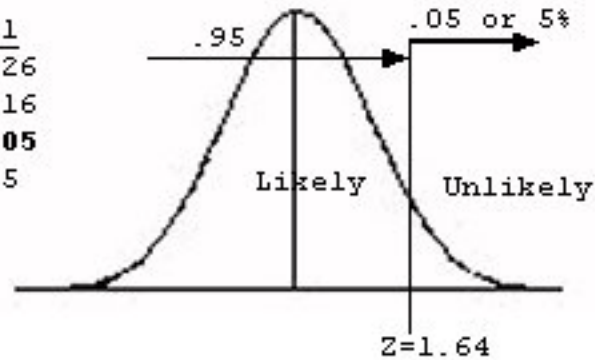
So, the likelihood of observing an IQ of 130 or more is .0228. Since the probability is less than 5% or .05 we have to assume Neil comes from a different population than the one we know about (the general population of humans).

### Critical Value

Critical Values are a way to save time with hypothesis testing. We don't really have to look up the probability of getting a particular value in order to verify it is less than 5% likely. The reason for this fact is that the z-score that marks the point where a value becomes unlikely does not change on the z-scale. That is, there is only one z-score at that is 5% likely. Any z-score beyond that point is less than 5% likely. Thus, I don't have to look up each particular area when I compute my z-score. Instead I only have to verify that the z-score I computed is more extreme than the one that is 5% likely. So, we can stop once we compute the z-score without reference to the z-table.

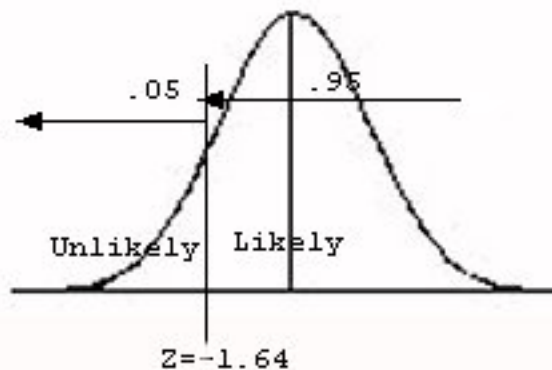
What z-score will be exactly 5% likely for any population? This is the z-score we will make comparisons against. Use the z-table to determine the z-score that cuts off the top 5% of scores. Finding the critical value is important, and will be one of the steps that must be performed anytime we conduct a hypothesis test. Since we will be doing hypothesis testing from this point on, many points on subsequent exams will come from just knowing the critical value.

Z	Body	Tail
1.62	.9474	.0526
1.63	.9484	.0516
<b>1.64</b>	<b>.9495</b>	<b>.0505</b>
1.65	.9505	.0495



So,  $Z = 1.64$  is the critical value. Any value more extreme than 1.64 is unlikely, and all other events will be likely for the known population. If  $Z=2.1$ , or  $Z=1.88$  you conclude that the value is unlikely, and so must be part of a different population. If  $Z = 1.5$ , or  $Z = 0.43$  you conclude that the value is likely, and so must be part of the known population.

Note that we were only working on one side of the distribution in the above problem. If we were interested in a value that was below the mean, instead of above it, then we would flip our decision line to the other side. Since the distribution is symmetric, the numbers will not change.



### Alpha

Alpha is the probability level we set before we say a value is unlikely for a known population. The critical value we just found is only one that we will use. It assumes that a value must be less than 5% likely to be unlikely, and therefore part of a different population. Alpha was .05 ( $\alpha = .05$ ) for that example. Sometimes researchers want to be very sure before they decide a value is different. Thus, we will also use an alpha level of .01 or 1% as well. If alpha is .01, then a value must be less than 1% likely before it is said to be unlikely for a known population. If alpha is 1% then the critical value will be different than the one we found above. Alpha is given in every problem, but you must use that information to determine the critical value.

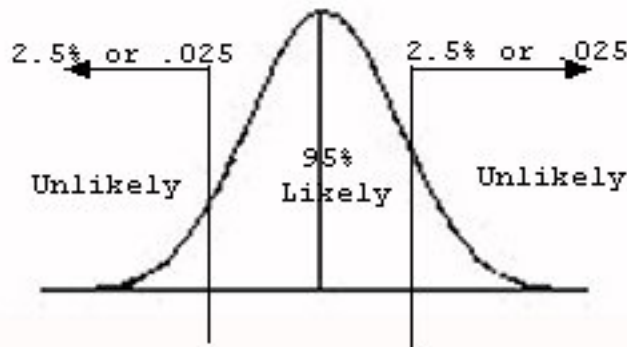
What z-score will be exactly 1% likely for any population? This is the z-score we will make comparisons against when alpha is set to 1% ( $\alpha = .01$ ). Use the z-table to determine the z-score that cuts off the top 1% of scores just like the last example. Use the Tail column and find .01. You could also look in the Body Column and find .99. We

will use  $Z = 2.33$  when Alpha is set to the 1% level. Also note that when we are interested in determining if values below the mean are unlikely our critical value will be negative.

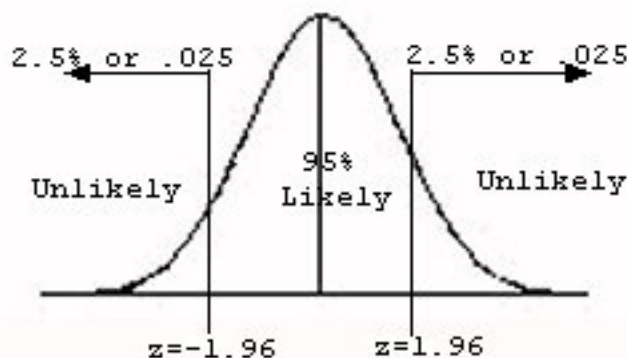
### One-Tail versus Two-Tail Tests

Another factor that will affect our critical value is whether we are performing a one or a two-tail test. The critical values we have looked at so far were for one-tail tests because we were only looking at one “tail” of the distribution at a time (either on the positive side above the mean or the negative side below the mean). With two-tail tests we will look for unlikely events on both sides of the mean (above and below) at the same time. I will discuss how to determine if a problem is a one or a two tail test in a later lesson, but let’s go ahead and find the critical values for two-tailed test the same way we did the one-tail tests above.

Let’s begin with an alpha level of 5%. We still want 5% of our events to be “unlikely” and 95% of our events to be “likely” for the known population. Now, however, we want to be looking for “unlikely” events in both directions at the same time. So, we will split the “unlikely” block into two parts, each half the total 5% area.



What z-scores will then mark the middle 95% of our distribution? You will have to look up an area of .025 in the Tail Column of the z-table.



The process for finding the two-tail critical values when alpha is set to .01 is the same. This time we will want 99% of our values in the middle, leaving only .005 or half of one percent on each side. Can you find the critical value on the z-table (answer:  $z = 2.58$ ).

So, we have learned four critical values.

	1-tail	2-tail
$\alpha = .05$	1.64	1.96/-1.96
$\alpha = .01$	2.33	2.58/-2.58

Notice that you have two critical values for a 2-tail test, both positive and negative. You will have only one critical value for a one-tail test (which could be negative).