Lesson 8
Probability

<u>Outline</u>
Probability of an Event
Probability of Single Events
Probability of Multiple Events
-without replacement
-mutually exclusive events
Conditional Probability

Probability of an Event

There are three classes of events:

Impossible Events--------------Possible Events-----------Certain Events
P = 0 P = 0 to 1 P = 1

The probability (P) of an impossible event is zero because there is zero chance of it happening. A certain event has a probability of one or 100% because it will always happen. Most of the events we will be interested in are possible events. These probabilities will always have a value between zero and one. Always leave your answer in decimal form, instead of fractional form. Simply divide out any fraction you have by dividing the top number by the bottom number. So, ¼ is 0.25.

A simple experiment we could run to examine probabilities is to roll a six-sided die. What is the probability of rolling a '5' on a single die roll? Most people know it is 1/6 or .167 because there is only one side that is a '5' and there are six sides on the die. You can determine the probability of any even in this manner.

$$\text{Probability } (P) = \frac{\#\_items\_in\_criteria}{Total\_\#\_items}$$

So, we were only looking for one side in the last problem, out of a total of six sides.

Another experiment we could use to look at probabilities is drawing cards from a standard deck. If I draw a card from a deck of cards what is the probability it is a heart?

$$\text{P}\,(Heart) = \frac{13}{52} = \frac{1}{4} = 0.25$$

Note that a standard deck of cards has 52 cards with 13 hearts/13 clubs/13 diamonds/13 spades. Since diamonds and hearts are red cards and the rest are black, there are 26 red and 26 black cards.

Probability of Single Events

An individual event is a single event. With single events we are measuring the likelihood of a one thing happening. We might be interested in different outcomes, but we are still just going to roll the die once or draw a single card from a deck. For example, what is the probability that I roll a '5' or a '6' on a single die roll or P (5 or 6)? With single events you will see this "or" connector and you will add the two individual probabilities. So:

P (5 and 6)
= 1/6 + 1/6
= .167 + .167
= .334

What is the probability I draw a Heart or Club with a single draw from a standard deck of cards? P (Heart or Club)

P(Heart or Club)
= P(Heart) + P(Club)
= 13/52 + 13/52
= .25 + .25
= .5

Probability of Multiple Events

With multiple events we will be interested more than one outcome be realized. So, we will roll the die more than once or draw more than one card from a deck. For example, what is the probability of rolling a '5' and a '6' on two die rolls. To get both a five and a six I will have to roll the die more than once. When you see this "and" connector you will multiply individual probabilities.

P (5 and 6)
= P(5) * P(6)
= 1/6 * 1/6
= .167 * .167
= .028

We will only be dealing with independent events in this section, or events that do not affect the outcome of other events. With the card experiment, then, we will not look at multiple draws where one draw could affect the probability of a separate event. For example, what is the probability of drawing a Heart and a Club from standard deck?

P(Heart and Club)
= P(Heart) * P(Club)
= 13/52 * 13/52
= .25 * .25
= .062

Without Replacement

Although we will focus on independent events like the last example, we will also consider what happens to probabilities in situations in which there is no replacement. The above examples assumed that once we drew a card from the deck that it was replaced before another draw was made. Notice that when figuring how many total events there were we used 52 every time because we assumed each draw was from a fresh deck. If the problem, however, specifies that there is no replacement then we must take this into account when figuring the probabilities.

For example, what is the probability of drawing a Heart and a Club from a deck without replacement? When we count how many cards are left for the Club draw, there will be one less card in the deck because we already had to draw the Heart from the deck. Thus:

P(Heart and Club)
= P (Heart) * P (Club)
= 13/52 * 13/51
= .25 * .255
= .064

We might also have to subtract a value from the numerator as well as the denominator. Try to find the probability of drawing three red cards from a deck without replacement. (Answer: 0.1176)

Mutually Exclusive Events

Mutually exclusive events are events that cannot happen together. For example, being a freshman and a sophomore are mutually exclusive. You are either one or the other but not both. For mutually exclusive events the probability the two events will occur together must always equal zero.

Conditional Probability

With conditional probabilities we will consider the probability of an event given that some other event has already happened. Thus, these are not independent events, and the rules we learned above will not apply. For these problems frequency data (or counts) will be given in a contingency table. This table will display the frequencies for different combinations of events. For example, consider the probability of having a computer or not, and living the U.S. or elsewhere.

	In U.S.	Not in U.S.
Computer	30	15
No Computer	10	20

Before we consider conditional probabilities, let's look at some of the types of questions we have already examined. For example, what is the probability that choosing someone from our sample will yield a person with a computer?
To answer this question we will need to add up the total for each row and column in the table:

	In U.S.	Not in U.S.	Total
Computer	30	15	45
No Computer	10	20	30
total	40	35	

Since there are a total of 45 people in our sample with a computer out of 75 total people, there is a 0.6 probability that a random draw will yield a person with a computer. Now find the probability that a random draw will yield someone with a computer that is living in the U.S. Instead of looking in the total column for this type of problem, we will use one of the original values. There are 30 people living in the U.S. that also have a computer. So, 30 out of the total of 75 people or 0.4 live in the U.S. and have a computer. For a single event in a table like this one, use the values in the margin or the totals, and divide by the total number in the sample. For a combined event, use the original table values out of the total.

For conditional probabilities we will restrict our sample to those items given to have already have happened. For example we might know the probability of having a computer and living in the U.S. for the entire sample. We could make this conditional by saying what is the probability of having a computer given that we know the person is living in the U.S.? With the second question we are not asking the probability of picking a person at random from the total, but instead we are restricting our sample to just those that live in the U.S. For conditional probabilities the total or denominator is the value given. For this example it is the total for those living in the U.S. or 40. We want to know what proportion have a computer out of these 40 people living in the U.S. Since 30 of those in the U.S. have a computer out of 40 the probability is 30/40 = .75

In this same example what is the probability someone does not live in the U.S. given they have a computer? We can write:

P (not in U.S. | computer)

Where the first probability is the one we are interested in, the vertical line means "given" and the second probability "computer" is what is given. Again, our new total is those with a computer or 45. We want to know the proportion out of these that are not in the U.S. or 15. So, the conditional probability is 15/45 = .33