

Lesson 7 Z-Scores and Probability

Outline

Introduction

Areas Under the Normal Curve

Using the Z-table

Converting Z-score to area

-area less than z/area greater than z/area between two z-values

Converting raw score to Z-score to area

Converting area to Z-score to raw score

Introduction/Area Under the Curve

Please note that area, proportion and probability are represented the same way (as a decimal value). Some examples require that you convert the decimal value to a percentage. Just move the decimal place to the right two places to turn the decimal into a percentage.

Start this section by reviewing the first two topics in the above outline on the web page.

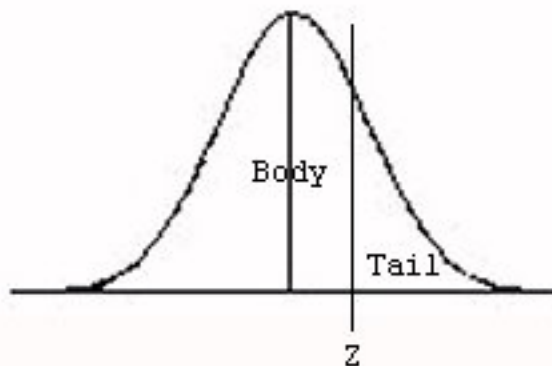
Find the Z-score animated demonstrations or click here

<http://faculty.uncfsu.edu/dwallace/sz-score.html>

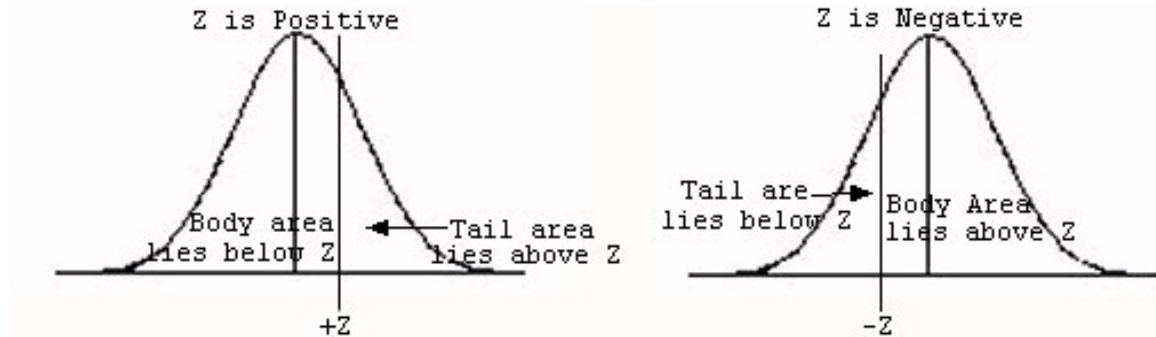
Using the Z-table

Knowing the number of standard deviations from the mean gives us a reliable way to know how likely a score is for a population. There is a table of z-scores that gives the corresponding areas or probabilities under the curve. You will need the z-table in Appendix B of your text for this discussion. See page A-24 through A-26 in your text.

The table shows the z-score in the left column, and then the proportion or area under the curve in the body, and finally there is a third column that shows the proportion or area under the curve in the tail of the distribution. Whenever we compute a z-score it will fall on the distribution at some point. The larger portion is the body, and the smaller portion is the tail.



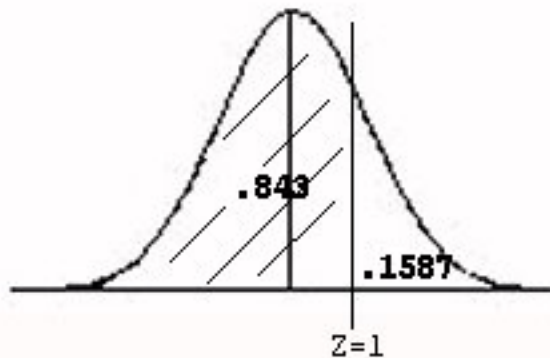
If the z-score is positive, then the body will be the area that is below that z-score, and the tail will be the area that is above that z-score. If the z-score is negative, then the body will be the area that is above that z-score, and the tail will be the area that is below that z-score.



Converting a Z-Score to an Area

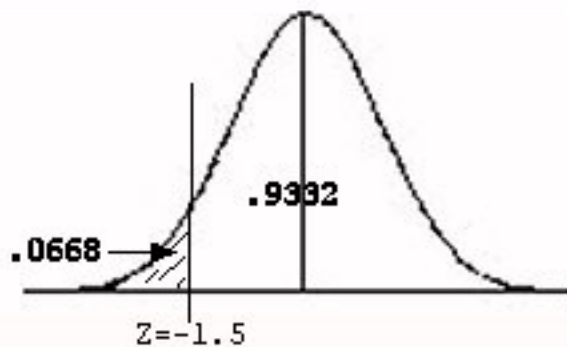
Finding areas below a z-score

What area lies below a z-score of +1? If we look this z-score up on the table we find that the area in the body is .8413 and the area in the tail is .1587. Since the z-score is positive and we want the area below the z-score, then we will want to look at the body area. So, .843 is the proportion in the population that have a $z = 1$ or less.



Now let's consider the situation if the z-score is negative. What area lies below a z-score of -1.5? You will not find negative values on the table. The distribution is symmetric, so if we want to know the area for a negative value we just look up the positive z-score to find the area. We will use a different column on the table, and that is why we must consider whether z is positive or negative when using the table.

If we look this z-score (1.5) up on the table we find that the area in the body is .9332, and the area in the tail is .0668. Since the z-score is negative and we want the area below that point we will be using the tail area. So, .0668 is the proportion in the population below a z-score of -1.5.



Finding areas above a z-score

The process for this type of problem is the same as what we have already learned. The only difference is in which column we will be using to answer the question. What area lies above a z-score of +1?

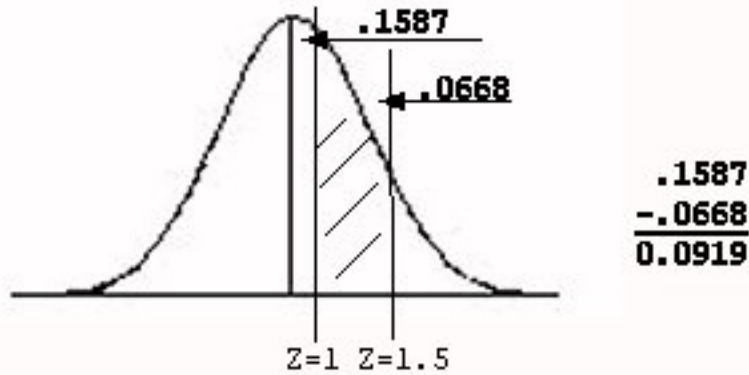
If we look this z-score up on the table we find that the area in the body is .8413 and the area in the tail is .1587. Since the z-score is positive and we want the area above the z-score, then we will want to look at the tail area. So, .1587 is the proportion in the population that have a $z = 1$ or more.

Now let's consider the situation if the z-score is negative. What area lies above a z-score of -1.5? Again, you will need to look up the positive z-value for 1.5. If we look this z-score up on the table we find that the area in the body is .9332, and the area in the tail is .0668. Since the z-score is negative and we want the area above that point we will be using the body area. So, .9332 is the proportion in the population above a z-score of -1.5.

Finding areas between two z-scores

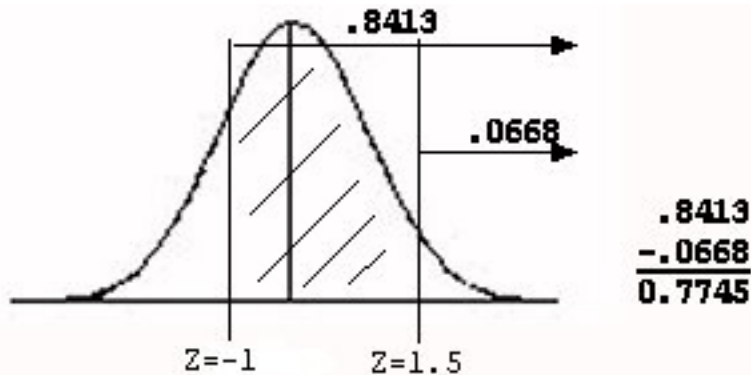
When we have two different z-scores and want to find the area between them, we first must consider if both values are on the same side of the mean, or if one value is positive and the other negative. For our table, if the values are either both positive z's or both negative z's, we can find the tail area for both z-scores and subtract the two areas. You could just as easily find the two body areas for both z-scores and subtract them as well.

For example, what is the area between $Z = 1$ and $Z = 1.5$? Since both scores are positive and we want the area between them, we will look up the tail area and subtract the two table values. Note that you never subtract z-scores, only areas from the table.



On the other hand, if you have one positive and one negative z-score then you must use the body area for either one of the z-scores, and the tail area for the other. Once you get the two areas off the table, then you subtract the two areas.

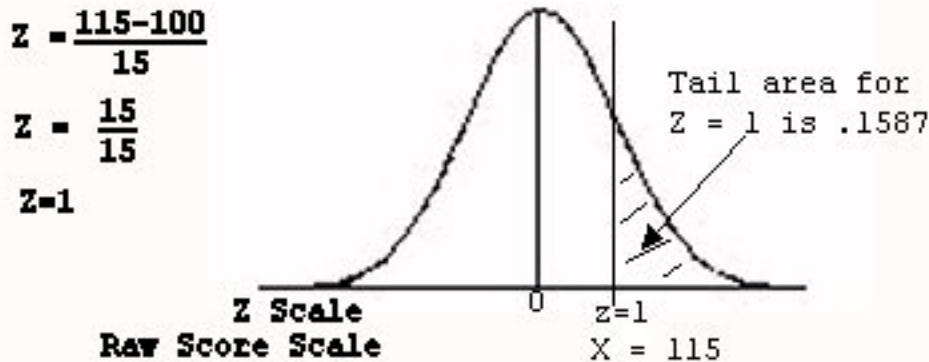
For example, what is the area between $Z = -1$ and $Z = 1.5$? Since one score is positive and the other negative, the area we are looking for will cross the mean. Use the body area for one value and the tail area for the other. Once you get these values off the table, subtract them to find the area in between.



Converting a Raw Score to a Z-score and then into an Area

These problems are exactly the same as the others we have been working. You must still find areas above/below/and between two z-scores, but now you must first compute the z-value using the z-formula before using the table.

For example let's look at IQ scores for the population with a mean of 100 and standard deviation of 15 ($\mu = 100, \sigma = 15$). What proportion of the population will have an IQ of 115 or less? I first must compute Z. It is equal to $z = 1$. Now the question becomes what proportion of scores lie above $z = 1$?



Please review other examples of this type of problem on the web-page. Find the link to Z-scores and probability or click here <http://faculty.uncfsu.edu/dwallace/sz-score2.html>

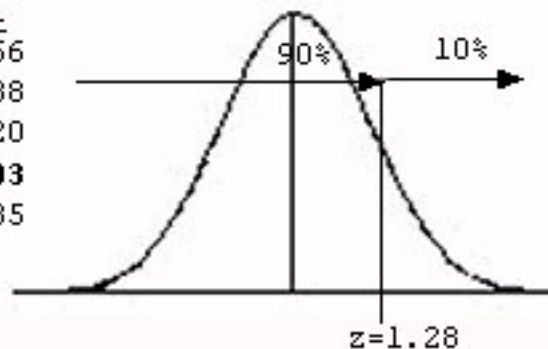
Converting an Area to a Z-score and then into a Raw Score

For these problems we will be doing the same process we have been doing, but everything will be done in reverse order. We will start with a given area or proportion. You then use the z-table to find the area. However, when you use the z-table for these problems you must look up the area in either the body or tail column and then trace it back to find the z-score. Once we get the z-score we will plug in the values we know and solve for X in the z-score equation.

For example, IQ scores for the population with a mean of 100 and standard deviation of 15 ($\mu = 100, \sigma = 15$), what score cuts off the top 10% of the distribution? Notice that these questions are always asking what the score is for a certain point. We are solving for X now. Prior examples were all asking for an area or proportion.

The first step to solving this type of problem is to find the Z-score. We won't be computing z, but instead finding it from the table. Since we want the top 10%, we will be looking for the area on the table where the tail is .10 and the body is .90. You can look in either column, and it might help to draw the distribution in order to be sure you are using the right column. Make sure you are not using the z-column at this point.

Z	Body	Tail
1.25	.8944	.1056
1.26	.8962	.1038
1.27	.8980	.1020
1.28	.8977	.1003
1.29	.9015	.0985



We find the z-score that leaves the tail at the top of the distribution equal to .10 is Z = 1.28. Always use the z-score closest to the area of interest. Once we have that number,

we can plug in what we know into the z-formula and solve for X. Alternatively, if you have trouble with algebra, you can use the following formula:

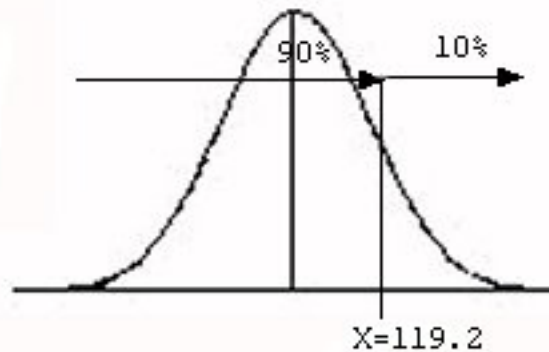
$$X = Z\sigma + \mu$$

$$1.28 = \frac{X - 100}{15}$$

$$X - 100 = 1.28(15)$$

$$X - 100 = 19.2$$

$$X = 119.2$$



Special note for values in the lower 50% of the distribution: Whenever we want to find a z-score for a value below the mean, we must remember to make the value negative. Recall that the z-table only gives positive z-values. If the value is below the mean, then you must remember to insert the negative sign before doing the computation. For example, if I were looking for the IQ score for the bottom 10% of the distribution, in the above example, then I would have look up a tail area of .10 or body area of .90. The z-score we need is -1.28 even though the table shows only the positive value.

Please review other examples of this type of problem on the web-page. Find the link to Z-scores and probability or click here <http://faculty.uncfsu.edu/dwallace/sz-score2.html>